

APPLICATION FOR UNITED STATES LETTERS PATENT

FOR

**Multi-Variate Data and Related Presentation and Analysis**

Inventor(s):

**Richard A. Van Koningsveld**

Prepared by:

**COLUMBIA IP LAW GROUP, PC**

**Express Mail Label No.: EV051081895US**

**Date of Deposit: December 31, 2001**

## **Multi-Variate Data and Related Presentation and Analysis**

### **BACKGROUND OF THE INVENTION**

5

#### **1. Field of the Invention**

The present invention relates to the field of data processing. More specifically, the present invention relates to data processing techniques associated with multi-variate data and related presentation and analysis.

10

#### **2. Background Information**

Many problems and/or applications involve the processing and presentation of multi-variate data and/or multi-dimensional functions, especially in the areas of academic research and other applied domains where mathematical

15

equations/quantitative data are used in presentations or decision making. Examples of such academic research and other applied domains include but are not limited to the disciplines of sociology, psychology, geology, physics, meteorology, biology, marketing, and other complex scientific disciplines.

A number of multi-variate presentation and analysis techniques are known in the art. However, all of these prior art techniques suffer from at least the disadvantage of not being able to effectively convey the essence and/or relations of high dimensional data, in particular, not being able to ordinate the essence and/or relations of quantitative information of more than three dimensions in a spatially and/or cognitively coherent way.

25

Thus, an improved presentation and/or analysis technique for handling and/or dealing with multi-variate data and/or multi-dimensional function is desired.

## SUMMARY OF THE INVENTION

5 A recursively partitioned/nested geometric structure is employed to graphically present and/or facilitate analysis of multi-variate data, including multi-dimensional functions. Visual attributes are associated with appropriate ones of the regions within the geometric structure, in accordance with determined graphing values for the regions.

10 In one embodiment, the data are normalized, comprising relative coordinate values, and encoded into polynary strings. Processing is advantageously performed using the polynary strings.

15 In one embodiment, a graphing function and a data format converter are incorporated with the teachings of the present invention to practice the present invention.

20 In one embodiment, a processor based apparatus is equipped with the graphing function and the data format converter.

### BRIEF DESCRIPTION OF DRAWINGS

The present invention will be described by way of exemplary embodiments,  
5 but not limitations, illustrated in the accompanying drawings in which like references  
denote similar elements, and in which:

**Figure 1** illustrates an overview of the present invention, in accordance with  
one embodiment;

**Figures 2a-2d** illustrate the sequential organization schematic of one  
10 example recursively partitioned/nested geometric structure based on a line segment  
that may be used by the graphic function of **Fig. 1** to practice the present invention;

**Figures 3a-3c** illustrate the sequential organization schematic of another  
example recursively partitioned/nested geometric structure based on a triangle that  
may be used by the graphic function of **Fig. 1** to practice the present invention;

**Figures 4a-4b** illustrate the sequential organization schematic of yet another  
example recursively partitioned/nested geometric structure based on a pentagon  
that may be used by the graphic function of **Fig. 1** to practice the present invention;

**Figure 5** illustrates an example recursively partitioned/nested geometric  
structure based on different geometric primitives (a heptagon and a circle) that may  
20 be used by the graphic function of **Fig. 1** to practice the present invention;

**Figure 6** illustrates another example recursively partitioned/nested geometric  
structure based on different geometric primitives (a triangle and a circle) that may be  
used by the graphic function of **Fig. 1** to practice the present invention;

**Figure 7** illustrates the operational flow of the relevant aspects of the data  
25 format converter of **Fig. 1** for generating a polynary string encoding a set of relative  
coordinates, in accordance with one embodiment;

**Figure 8** illustrates the operational flow of the relevant aspects of the data format converter of **Fig. 1** for generating the relative coordinates of a polynary string, in accordance with one embodiment;

**Figure 9** illustrates the overall operational flow of the graphing function of

5 **Fig. 1**, in accordance with one embodiment;

**Figure 10** illustrates the operational flow of the relevant aspects of the graphing function of **Fig. 1** for presenting a collection of multi-variate data and facilitating their analysis, in accordance with one embodiment;

**Figure 11** illustrates the operational flow of the relevant aspects of the graphing function of **Fig. 1** for presenting a multi-dimensional function, and facilitating its analysis, in accordance with one embodiment; and

**Figure 12** illustrates an internal component view of a processor based system suitable for use to practice the present invention, in accordance with one embodiment.

15

## DETAILED DESCRIPTION OF THE INVENTION

The present invention, as summarized earlier, includes data processing

5 presentation and analysis techniques for multi-variate data and/or multi-dimensional function. In the description to follow, various aspects of the present invention will be described, specific operational flows and configurations will be set forth. However, the present invention may be practiced without some of these specific details. In other instances, well-known features are omitted or simplified in order not to obscure

10 the present invention.

The description will be presented in terms of operations performed by a processor based device, using terms such as data, functions, format conversions, determining, associating, and so forth, consistent with the manner commonly employed by those skilled in the art to convey the substance of their work to others

15 skilled in the art. As well understood by those skilled in the art, the quantities take the form of electrical, magnetic, or optical signals capable of being stored, transferred, combined, and otherwise manipulated through mechanical, electrical and/or optical components of the processor based device. Moreover, the term processor includes microprocessors, micro-controllers, digital signal processors, and the like, that are

20 standalone, adjunct or embedded.

Various operations will be described as multiple discrete steps in turn, in a manner that is most helpful in understanding the present invention, however, the order of description should not be construed as to imply that these operations are necessarily order dependent. In particular, these operations need not be performed

25 in the order of presentation.

The description repeatedly uses the phrase “in one embodiment”, which ordinarily does not refer to the same embodiment, although it may. The terms “comprising”, “including”, “having”, and the like, as used in the present application, are synonymous.

5

### Overview

Referring now to **Figure 1**, wherein a block diagram illustrating an overview of the present invention, in accordance with one embodiment, is shown. As illustrated, for the embodiment, graphing function **102** and data format converter **104**,

10 incorporated with the teachings of the present invention are provided to facilitate presentation and analysis of multi-variate data **106** of a multi-dimensional data space or multi-dimensional functions (not shown). More specifically, graphing function **102** is equipped to advantageously present multi-variate data **106** and/or multi-dimensional functions employing recursively partitioned and/or nested geometric structures, which as will be apparent from the description to follow, rendering high-dimensional data more comprehensible. Further, for the embodiment, graphing function **102** advantageously processes the multi-variate data or variable values of a multi-dimensional function encoded in the polynary string form of the present invention, enabling the processing, presentation, and facilitation of analysis to be  
15 accomplished more efficiently. For the embodiment, data format converter **104** is provided to assist graphing function **102** in converting multi-variate data **106** and variable values of a multi-dimensional function between a relative coordinate form, and the polynary string form of the present invention.

20

25 Multi-variate data **106** and the multi-dimensional functions represent a broad range of such data and functions. Examples include but are not limited to data and functions commonly encountered in problems and/or studies in the disciplines of

sociology, geology, physics, biology, and other complex scientific disciplines of the like.

Further, the present invention contemplates that graphing function **102** and data format converter **104** may be jointly realized in a standalone product or part of a  
5 larger product, or separately realized as two standalone products or part of two respective larger products, or combinations there of.

#### Recursively Partitioned/Nested Geometric Structure

**Figures 2a-2d** illustrate the sequential organization schematic of an example

10 recursively partitioned and/or nested geometric structure may be used by graphic function **102** to present and facilitate analysis of “two part” multi-variate data **106** and/or “two dimension” functions. As illustrated, geometric structure **202d** is formed by recursively partitioning geometric structure **202a**, a “line” segment. For the example, line segment geometric structure **202a** is first partitioned into two portions,  
15 top portion **204a** and bottom portion **204b**, to form geometric structure **202b**. Then, the two portions **204a** and **204b** of geometric structure **204b** are respectively partitioned in like manner into, portions **206a**, **206b**, **206c** and **206d**, to form geometric structure **202c**. The four portions **206a-206d** in turn are respectively partitioned in like manner into portions **208a-208h** to form geometric structure **202d**.

20 As those skilled in the art would appreciate, geometric structure **202d** may also be described as being formed by recursively nesting geometric structure **202a** into the successively formed geometric structures **202b** and **202c**. For example, geometric structure **202a** is first nested into a top and a bottom portion of itself to form geometric structure **202b**. Then, geometric structure **202a** is again nested into a top  
25 and a bottom portion of each of the two previously inserted or nested portions to form geometric structure **202c**. Likewise, geometric structure **202a** is again nested into a

top and a bottom portion of each of the four previously inserted or nested portions to form geometric structure **202d**.

The recursive partition and nesting process, in theory, may continue indefinitely. However, as those skilled in the art would appreciate, at certain level,

5 depending on the granularity of the display device (such as, the amount of pixels available), the usefulness of the additional details of the recursively partitioned/nested geometric structure may be limited.

As labeled, the successively formed portions may also be referred to as regions A and B **204a** and **204b**, regions AA, AB, BA and BB **206a-206d**, and regions

10 AAA, AAB, ABA, ABB, BAA, BAB, BBA, and BBB **208a-208h**, or considered as regions corresponding to these labels. As will be readily apparent from the description to follow, the inherent ability to reference regions of such a recursively partitioned/nested geometric structure, allows employment of the polynary strings of the present invention to efficiently and effectively accomplish the desired presentation 15 and analysis.

**Figures 3a-3c** illustrate the sequential organization schematic of another example recursively partitioned and/or nested geometric structure may be used by graphic function **102** to present and facilitate analysis of “three part” multi-variate data

106 and/or “three dimension” functions. As illustrated, geometric structure **302d** is 20 formed by recursively partitioning geometric structure **302a**, a triangle. For the example, triangle or geometric structure **302a** is first partitioned into three portions, top-right portion **304a**, top-left portion **304b**, and bottom portion **304c**, to form geometric structure **302b**. Then, the three portions **304a-304c** of geometric structure 25 **304b** are respectively partitioned in like manner into, at least three portions each, portions **306a-306j**, to form geometric structure **302c**. The nine portions **306a-306j** in turn are respectively partitioned in like manner to form geometric structure **302d**.

Again, as described earlier, geometric structure **302d** may be considered as being formed by recursively nesting geometric structure **302a** into the respective portions of the successively formed geometric structures **302b** and **302c**.

The recursive partition and/or nesting process of **Fig. 3** too may in theory be 5 continued indefinitely. Similarly, depending on the granularity of the display device, the usefulness of the additional details of the geometric structure may be limited.

Further, as labeled, the successively formed portions may be referred to as regions A, B and C **304a-304c**, regions AA, AB, AC, BA, BB, BC, CA, CB and CC 10 **306a-306j**, and regions AAA, AAB, AAC, ABA, ABB, ABC, ACA, ACB, ACC, BAA, BAB, BAC, BBA, BBB, BBC, BCA, BCB, BCC, CAA, CAB, CAC, CBA, CBB, CBC, CCA, CCB and CCC, or considered as regions corresponding to these labels.

**Figures 4a-4c** illustrate the sequential organization schematic of another example recursively partitioned and/or nested geometric structure may be used by graphic function **102** to present and facilitate analysis of “five part” multi-variate data 15 **106**, and/or “five dimension” functions. As illustrated, geometric structure **402c** is formed by recursively partitioning geometric structure **402a**, a pentagon. For the example, pentagon or geometric structure **402a** is first partitioned into at least five portions, in a clockwise direction, portions **404a**, **404b**, **404c**, **404d** and **404e**, to form geometric structure **402b**. Then, the five portions **404a-404e** of geometric structure 20 **404b** are respectively partitioned in like manner into at least five portions each, to form geometric structure **402c**.

Again, as described earlier, geometric structure **402c** may be considered as being formed by recursively nesting geometric structure **402a** into the respective portions of the successively formed geometric structures, such as geometric structure 25 **402b**.

As the earlier examples, the recursive partition and nesting process may also be continued indefinitely.

As labeled, the successively formed portions may also be referred to as regions A, B, C, D and E **404a-404e**, regions AA, AB, AC, AD, AE, BA, BB, BC, BD, 5 BE, CA, CB, CC, CD, CE, DA, DB, DC, DD, DE, EA, EB, EC, ED and EE, or considered as regions corresponding to these labels.

**Figure 5** illustrates yet another example of a recursively partitioned/nested geometric structure may be used by graphing function **102** for presentation and analysis of “seven part” multi-variate data **106**, and/or “seven-dimension” functions.

10 The example geometric structure may be formed by recursively partitioning a heptagon geometric structure, as described earlier for the earlier examples, with the modification that the resulting regions are denoted using a different geometric primitive, i.e. the circle.

Thus, it can be seen from these descriptions, the geometric structures formed 15 in the above described recursively partition or nesting process remain fully scalable, applicable to very high dimensional data. In other words, Recursively partitioned geometric structures suitable for use to practice the present invention need not always be formable in a parallel recursive nesting process employing the same geometric primitive as in the earlier described example cases.

20 **Figure 6** illustrates another one of such example recursively partitioned/nested geometric structure employing different geometric primitives. Geometric structure **600** may be formed by first partitioning a triangle geometric structure into three parts . The three parts in turn are recursively partitioned in like manner, creating the regions shown, but depicting the resulting regions using a different geometry primitive, e.g. a 25 circle, to form the final geometric structure **600**. Again, the process may continue to form other further partitioned geometric structures. However, it is readily apparent

that the resultant geometric structure, such as geometric structure **600** is not formed by recursively nesting the same geometric structure/triangle into itself or the previously nested structures.

As will be readily apparent from the description to follow, any of these

5 geometric structures or other geometric structures formed in a like manner, i.e. through recursively partitioning, recursive nesting, or both, employing the same or multiple geometric primitives, may be employed to practice the present invention to facilitate presentation and analysis of multi-variate data **106** and/or multi-dimensional functions. Geometric structures formed in the described manner are scalable to even  
10 the very high dimensional data and/functions encountered in many complex problems and/or applications.

#### Relative Coordinates and Polynary Strings

As alluded to earlier, in various embodiments, to facilitate efficient operation of

15 graphing function **102**, multi-variate data **106** are advantageously normalized into relative coordinates, and encoded into polynary strings of the present invention. A normalized data comprising relative coordinates is a data vector having multiple non-negative parts that numerically sum to the value of "1". A polynary string comprises a sequence of symbols. The length of the sequence (K) is determined by the precision  
20 desired. For example, a particular mixed fruit salad having 51% apples, 17% bananas, and 32% cherries comprises the relative coordinates of {.51, .17 and .32}, and as will be shown later, may be represented by the polynary string of ACACB, having a sequence length of 6, i.e. 6 occurrences of the symbol A, B or C, where the symbols A, B and C correspond to the fruit Apple, Banana and Cherry respectively.

25 **Figure 7** illustrates the operation flow of the relevant parts of data format converter **104** for generating a polynary string to represent a normalized multi-variate

data comprising a plurality of relative coordinates, in accordance with one embodiment. For each normalized multi-variate data comprising a plurality of relative coordinates, for the embodiment, data format converter **104** first computes a working constant (F) by dividing (N – 1) by N, where N equals the number of relative

5 coordinates, block **702**. Next, data format converter **104** computes a current version of a working variable (G) by subtracting the constant F from 1, block **704**.

For example, for the example fruit salad mentioned earlier, N equals 3, therefore F equals 2/3 or 0.6667. Further, G initially equals 1/3 or 0.3333.

Then, data format converter **104** determines which of the relative coordinates 10 has the largest coordinate values, block **706**. Upon so determining, data format converter **104** selects the symbol corresponding to the current relative coordinate having the largest relative coordinate value, and includes the selected symbol as part of the polynary string, block **708**. For the embodiment, the selected symbol is appended to the polynary string (initially, a null string).

15 Again, using the earlier mentioned fruit salad as an example, for the first iteration, the relative coordinate of Apple would be determined as being the highest relative coordinate value (0.51), accordingly, the corresponding symbol A would be selected as the first symbol of the polynary string.

Next, data format converter **104** determines if the desired level of precision, i.e. 20 the desired length of the sequence (K), has been reached, block **710**. If the desired level of precision/length has not been reached, data format converter **104** decrements the largest relative coordinate in a predetermined manner, for the embodiment, by the current value of the working variable G, block **712**. In like manner, data format converter **104** also decrements the working variable G in a 25 predetermined manner, for the embodiment, by the value of  $G^*(1 - F)$ , block **714**. In

other words, G is newly set to  $G^*F$  (as  $G - G^*(1 - F)$  equals  $G^*F$ ). Thereafter, the process returns to block 706 and continues from there as earlier described.

Referring again to the earlier described example salad, thus upon selection of the symbol A as the first symbol of the polynary string, data format converter 104

5 decrements the highest relative coordinate value, which is Apple, with the value of 0.51, by the initial value of G (i.e. G(1)), which as described earlier is 0.3333.

Therefore, after the decrement, the relative coordinate value of Apple is now .1767 (.51 – 0.3333). Further, the new current value of G, i.e. G(2), is 0.2222.

For the second iteration, the relative coordinate value of Cherry will be

10 determined as the highest relative coordinate value 0.32 versus 0.1767 for Apple and 0.17 for Banana. Accordingly, the corresponding symbol B is selected as the next symbol for inclusion in the polynary string. Upon selection, the relative coordinate value of Cherry is decremented by G(2) to 0.0978 (0.32 – 0.2222), and G(2) is decremented to 0.1481 generating G(3).

15 Eventually, the desired level of precision/length (K) is reached (yes branch of block 710), at such time, the process terminates, and the cumulative symbols are output as the polynary string of the normalized multi-variate data comprising a plurality of relative coordinates. The desired level of precision/length may be specified for data format converter 104 in any one of a number of known techniques,

20 e.g. as a parameter of an invocation function call.

Referring again to the fruit salad example, after six iterations, the polynary string (length = 6) is ACACBC, and the residual relative coordinate values for the three fruits are 0.285, 0.273 and 0.0319 respectively. The current value of G, i.e. G(7), is 0.0293. Assuming the polynary string of 6 symbols reaches the precision 25 level specified, the process terminates, and the sequence of symbols is returned.

Note that  $G(K+1)$  ( $G(7)$  in the fruit salad example) equals to the average residual relative coordinate values.

In some applications, the initial numeric values of the relative coordinate values of two or more variables of a data vector of a set of multi-variate data may be

5 the same. A tie breaking or precedence rule, such as left to right or right to left (with a data vector being perceived as a “horizontal” or “row” vector), or top to bottom or bottom to top (with a data vector being perceived as a “vertical” or “column” vector), may be consistently employed to supplement the earlier described process for generating a polynary string of the present invention. For example, an alloy

10 comprising equal number of parts of iron and carbon, has the relative coordinate value of 0.5 for both iron and carbon. Thus, the alloy may be represented by a polynary string of IC or CI, where I stands for iron, and C stands for carbon. Whether IC or CI is generated depending on the tie breaking rule consistently employed. If the first and second data tuples of the “row” data vector represent the content of iron and

15 carbon respectively, the polynary string IC will be resulted if the left to right precedent rule is employed. Alternatively, for the same data vector, the polynary string CI will be resulted if the right to left precedent rule is employed instead. The preference is application dependent. In alternate embodiments, other tie breaking or precedent rules may be employed to address the issue of equal relative contributions, as long

20 as the rule is consistently employed.

As alluded to earlier, depending on the type of graphing being performed, from time to time, graphing function **102** may need to recover the relative coordinate values of the normalized multi-variate data represented by a polynary string. **Figure 8** illustrates the operational flow of the relevant parts of data format converter **104** for

25 recovering the relative coordinates of a normalized multi-variate data represented by a polynary string. As illustrated, data format converter **104** selects one of the

variables, block **802**. Data format converter **104** identifies its symbol, e.g. from a data dictionary, and the positions the corresponding symbol appears in the polynary string, if any, block **804**. For the embodiment, by virtue of the manner polynary strings are generated, a contribution value is attributable to each position a symbol appears in

5 the polynary string. The contribution value equals to the then current value of the working variable  $G$ , or  $G(i)$ , at the time (i) a particular symbol is selected and included as part of the polynary string. Thus, to recover the corresponding relative coordinate value for the symbol, data format converter **104** sums the positional values of the appearances of the symbol  $G(i)$ s, where  $i$  is less than  $K+1$ , and the terminal value of

10 the working variable  $G$ , i.e.  $G(K+1)$ , block **806**, and outputs the sum as the recovered relative coordinate, block **808**.

In one embodiment, if the corresponding symbol of the variable does not appear in the polynary string (i.e. its relative coordinate value was never the highest when compared to other relative coordinate values, for the precision/sequence length desired), the relative coordinate value of the variable is set to the average residual relative coordinate value, i.e. the terminal value of the working variable  $G(K+1)$ . In alternate embodiments, other application dependent manners, more or less precise, may also be employed to address the issue of un-represented variable of a data vector.

20 Next, data format converter **104** determines if the corresponding relative coordinate have been recovered for all variables of the data vector represented by the polynary string (i.e. whether  $N$  variables have been processed), block **810**. If not, the process returns to block **802** and continues from there, as earlier described. If yes, the process terminates.

25 Referring to the fruit salad example again, three variables or ingredients were encoded ( $N=3$ ). The symbol A for the first variable/ingredient Apple appears in

position 1 and 3 of the polynary string ACABCB, thus the relative coordinate value for Apple is recovered by adding the values of G1, G3 and G7, which are .3333, .1481 and .0293 respectively. The result is the original relative coordinate value for the first variable/ingredient Apple, 0.5107. The relative coordinate value for the second

5 variable/ingredient Banana may be recovered in like manner by summing G4, G6 and G7 (B appears in positions 4 and 6), which are 0.0988, 0.0439 and 0.0293 respectively. The result is the original relative coordinate value for the second variable/ingredient Banana, 0.1720. The relative coordinate value for Cherry may also be recovered in like manner by summing G2, G5 and G7 (C appears in positions 10 2 and 5), which are 0.2222, 0.0658 and 0.0293 respectively. The result is the original relative coordinate value for the third variable/ingredient Cherry, 0.3173.

### Graphing and Analysis

Figure 9 illustrates the overall operational flow of graphing function 102 for presenting multi-variate data, including functions with multi-dimensional variables, and facilitating their analysis, in accordance with one embodiment. In addition to being able to present either multi-variate data or functions with multi-dimensional variables, the embodiment is also able to support presentation of multiple sets of data/functions in a side-by-side or multi-page manner, as well as facilitate “zooming” into a subset of the data/function being presented.

As illustrated, upon requested, graphing function 102 proceeds to process a first set of multi-variate data or function, generating the display data associated with a recursively partitioned/nested geometric structure with visually differentiated structural parts for use to present the data/function (to be described more fully below), block 25 902. In one embodiment, the generation of the display data associated with a recursively partitioned/nested geometric structure includes the generation of the

center of each of the primitive elements of the geometric structure. In one embodiment, the coordinates  $(x_p, y_p)$  of the center of each of the primitive elements of the geometric structure for polynary strings representing  $N$  relative coordinate values, each consisting of a sequence of  $K$  ( $k = 1, 2, \dots, K$ ) symbols, are generated as follows:

5

$$X_p = X_c + R * \sum_{k=1}^K V(N, k) * C(N, m[L_k])$$

$$Y_p = Y_c + R * \sum_{k=1}^K V(N, k) * S(N, m[L_k])$$

where:

$(X_c, Y_c)$  are the coordinate values of the geometric structure's centroid;

10  $R$  is the radius from the geometric structure's centroid to an outermost vertex of the geometric structure (which together with  $(X_c, Y_c)$  defines the overall size of the employed geometric structure;

$V(N, k)$  is the value  $w * (1 - w)^{k-1}$  where  $w = 1/(1 + \sin(\pi/N))$ ;

15  $m[L_k]$  is the outer vertex number (1, 2, ...,  $N$ ) assigned to the  $k$ th symbol appearing in the string (e.g. if  $A = 1$ ,  $B = 2$ , and  $C = 3$  are assigned to the outer vertices of a triangle, as counted in a counter-clockwise order on the geometric structure, for the polynary string CABA,  $m[L_1]$  is set to 3,  $m[L_2]$  is set to 1,  $m[L_3]$  is set to 2, and  $m[L_4]$  is set to 1);

20  $C(N, m[L_k]) = \cos(a * \pi)$ ; and

$S(N, m[L_k]) = \sin(a * \pi)$  [where  $a = (5 * N - 4 * m[L_k]) / (2 * N)$ ].

In one embodiment, a series of  $K$  values for  $V(N, k)$  are computed once and stored once  $N$  is specified. Similarly, the  $N$  values for  $C(N, m[L_k])$  and the  $N$  values of  $S(N, m[L_k])$  can also be computed once and stored. These pre-computations advantageously simplify the computations for the centroids of the various primitive elements of an employed recursively partitioned/nested geometric structure, as each

location is principally the sum of products of fixed numbers that depend only on the  $m[L_k]$ 's of a given polynary string.

Continuing to refer to **Fig. 9**, next, graphing function **102** determines if the request is for multiple sets of data or functions, block **904**. If so, graphing function 5 **102** successively returns to block **902** to process the additional data sets or functions, and generate the display data associated with the corresponding recursively partitioned/nested geometric structure with visually differentiated structural parts for each of the requested graphing operations.

Eventually, having generated all the display data associated with the requested 10 graph or graphs, graphing function **102** presents the generated geometric structure or geometric structures as requested. That is, if the request is for multiple graphs, and the format is for a side-by-side presentation, graphing function **102** scales the generated geometric structures, such that they may all be presented within a display window. The geometric structures may be scaled to a point such that all geometric 15 structures are fitted with the display window, so they may all be viewed at the same time. Alternatively, the geometric structures may be scaled by a predetermined amount such that they may be viewed through scrolling in the appropriate directions. However, for the embodiment, the request may also specify that the geometric structures be rendered in multiple display pages and be viewed by the user “flipping” 20 through the display pages (through any one of a number of user interaction techniques known in the art, e.g. page up, page down, and so forth).

Thereafter, i.e. after rendering the generated geometric structure or structures, graphing function **102** waits for user analysis commands, block **908**. Various analysis commands may be supported. The extent of the support is application dependent.

25 For the embodiment, included among the analysis commands supported is a zooming command. Moreover, in conjunction with the zoom command, a degree of zoom

expressed in term of the most significant symbol value or values of the polynary strings may be specified. Thus, upon receipt of an analysis command, graphing function **102** determines if the inputted command is the zooming command, block **910**. If the inputted command is a conventional user interaction command, such as 5 scrolling left, scrolling right and so forth, the inputted command is handled in a conventional manner, block **914**.

However, if the inputted command is the zoom command, graphing function **102** re-renders the presentation, rendering only the applicable sub-structures in accordance with the degree of zoom provided in conjunction with the zoom 10 command, in an expanded manner, block **912**. That is, the data are presented as if the data outside the scope of the specified zoom have been removed, and the remaining data within the scope of the zoom are encoded without the most significant symbol or symbols corresponding to the zooming specification.

For example, consider a collection of multi-variate data having data vectors 15 represented by polynary strings of the form AAxxxx, ABxxxx, ACxxxx, BAxxxx, BBxxxx, BCxxxx, CAxxxx, CBxxxx, and CCxxxx (where x stands for any constituting symbol). For the embodiment, if a zooming command is received with the zooming specification of "A", the sub-structures or regions corresponding to the data vectors encoded in the form BAxxxx, BBxxxx, BCxxxx, CAxxxx, CBxxxx, and CCxxxx are 20 excluded, whereas the sub-structures or regions corresponding to the remaining data vectors encoded in the form AAxxxx, ABxxxx, ACxxxx, are re-rendered in an expanded manner, considering only the residual lesser significant symbols of the polynary strings after the leading most significant symbol "A". Accordingly, the user is able to efficiently zoom into the "super" region "A", and analyze only the data vectors 25 with the variable represented by the symbol "A" being the dominant contributing variable. Recall that the symbol corresponding to the variable having the highest

initial relative coordinate values is selected first for inclusion in forming the polynary string. Thus, the symbol “A” occupying the first position of a polynary string denotes the variable represented by the symbol “A” has the highest initial relative coordinate value, and accordingly the most dominant variable.

5        In various embodiments, the zooming specification may be any sequence of the constituting symbols of the polynary strings of the data, of length less than K.

In various embodiments, the zoom command may also specify whether the zooming is to be applied locally to a particular geometric structure only, to a plurality of selected geometric structures, or to be applied globally to all requested

10      presentations, thereby allowing isolated zooming on one of the rendered geometric structures or coordinated zooming on selected ones or all the rendered geometric structures. Further, zooming may be incrementally increased, e.g. by first specifying the most significant symbol value or values for the first zoom, then specifying the next most significant symbol value or values for the second zoom, and so forth.

15      In various embodiments, in conjunction with the zooming command, an “un-zoom” command is also supported. In response to each invocation of the “un-zoom” command, graphing function **102** reverses the last zooming operation performed. In yet other embodiments, in lieu of or in addition to a straight forward un-zoom, different granularity of zooming and un-zooming may be supported. For example, 20      zooming may be advanced into a “super” region by two most significant symbol values, e.g. AB, and then un-zooming may be facilitated for just one symbol value “B”. The resultant presentation would be the same as if the user has specified zooming by “A” in the first place.

Upon handling the inputted commands, except in the case of the exit or 25      terminate command, graphing function **102** returns to block **908**, and awaits input of the next analysis command. The process continues, until eventually an exit or

terminate command is provided. At such time, graphing function **102** closes down its displays and terminates.

**Figures 10-11** illustrate in further details, the operational flow of graphing function **102** for generating a recursively partitioned/nested geometric structure, in

5 accordance with one embodiment each. More specifically, **Fig. 10** illustrates the operational flow for presenting a set of multi-variate data, whereas **Fig. 11** illustrates the operational flow for present a function of multi-dimensional variables.

As illustrated, for the embodiment, to present a set of multi-variate data, graphing function **102** first selects a data vector, normalized and encodes the data

10 into a polnary string, block **1004**, if necessary. Slack variables may be introduced to effectuate the normalization. Additionally, associated data required for intermediate calculations to generate the display data associated with the recursively partitioned/nested geometric structure may be stored in an associated working data set.

15 Then, graphing function **102** identifies the region within the recursively partitioned/nested geometric structure, to which the data vector corresponds, using the polnary string, block **1006**. As can be seen, the process of identification, i.e. the corresponding region within the recursively partitioned/nested geometric structure to be employed, can be readily and easily accomplished through the polnary string.

20 Next, graphing function **102** determines a graphing value for the corresponding region, block **1008**, and associates an appropriate visual attribute to the corresponding region based at least in part on the determined graphing value.

In one embodiment, if the requested presentation is for a set of multi-variate data, the graphing values are the frequencies of occurrences of the various polnary strings. Thus, in block **1008**, the graphing value for the corresponding region is incremented accordingly. The graphing values, as alluded to earlier, may be

maintained e.g. in an associated working data set. In another embodiment, if the requested presentation is for an output variable and a set of multi-variate input variables, the graphing value is simply the corresponding output value. In yet other embodiment, depending on the request, the graphing values may involve other

5 processing of the multi-variate data, such as computing their averages and so forth. In like manner, these graphing values may be maintained e.g. in an associated working data set.

Any one of a number of visual attributes may be employed to differentiate the different graphing values. In one embodiment, different symbols, such as the “+”

10 sign, the “-“ sign, or other symbols, are employed to differentiate different ranges of the graphing values. In another embodiment, different colors are employed to differentiate different ranges of the graphing values. In yet other embodiments, other visual differentiators may be employed instead.

Upon so processing a data vector, graphing function **102** determines if all data vectors have been processed, block **1012**. If additional data vectors remain to be processed, graphing function **102** returns to block **1002**, and continues from there as earlier described. Eventually, there will no unprocessed data vector. At such time, graphing function **102** generates the recursively partitioned/nested geometric structure employing the visual attributes associated with appropriate ones of the

20 regions, thereby differentiating the regions and allowing the multi-variate data to be pictorially conveyed to a user, block **1014**.

To present a function, as illustrated in **Fig. 11**, graphing function **102** selects a region of the recursively partitioned/nested geometric structure, block **1102**, determines the relative coordinates of the data vector based on a polynary string corresponding to the region, block **1104**. Upon recovering the relative coordinates, graphing function **102** evaluates the function, using the recovered relative

coordinates, block **1106**. Then, graphing function **102** associates a visual attribute to the region, based on the computed output value for the function, block **1108**. As before, the computed values may be maintained e.g. in a working data set,

Graphing function **102** then determines if all regions have been processed,

5 block **1110**. If additional regions remain to be processed, graphing function **102** returns to block **1102**, and continues from there as earlier described. Eventually, there will no unprocessed region. At such time, graphing function **102** generates the recursively partitioned/nested geometric structure employing the visual attributes associated with the regions, thereby differentiating the regions and allowing the

10 function to be pictorially conveyed to a user, block **1112**.

#### Example Processor Based Device

Figure 12 illustrates an example processor based device/system suitable for use to practice the present invention in accordance with one embodiment. As shown, example processor based device/system **1200** includes one or more processors **1202** and system memory **1204**. Additionally, example device/system **1200** includes mass storage devices **1206** (such as diskette, hard drive, CDROM and so forth), input/output devices **1208** (such as keyboard, cursor control and so forth) and communication interfaces **1210** (such as network interface cards, modems and so forth). The elements are coupled to each other via system bus **1212**, which represents one or more buses. In the case of multiple buses, they are bridged by one or more bus bridges (not shown). Each of these elements performs its conventional functions known in the art. In particular, system memory **1204** and mass storage **1206** are employed to store a working copy and a permanent copy of the programming instructions implementing the graphing function **102** and data format converter **104** incorporated with the teachings of the present invention. The

permanent copy of the programming instructions may be loaded into mass storage **1206** in the factory, or in the field, through a distribution medium (not shown) or through communication interface **1210** (from a distribution server (not shown)). The constitution of these elements **1202-1212** are known, and accordingly will not be

5 further described.

In various embodiments, processor based device/system **1200** may be a wireless mobile phone, a palm sized personal digital assistant, a notebook computer, a set-top box, a desktop computer or a server. In alternate embodiments, the present invention may also be practiced on single processor systems, multi-  
10 processor systems or clusters of coupled systems, e.g. networked through private and/or public networks.

#### Advantages

Thus, it can be seen from the above description, presentation and analysis of  
15 multi-variate data, or functions of multi-dimensional variables are made easier and more comprehensible under the present invention. Further, the present invention is advantageously scalable to high dimensional data/variables.

#### Conclusion and Epilog

20 Thus, an improved method and apparatus for presenting and facilitating analysis of multi-variate data and/or functions of multi-dimensional variables has been described. While the present invention has been described in terms of the above illustrated embodiments, those skilled in the art will recognize that the invention is not limited to the embodiments described. The present invention can be practiced with  
25 modification and alteration within the spirit and scope of the appended claims. Thus,

the description is to be regarded as illustrative instead of restrictive on the present invention.